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RAINFALL INTERPOLATION.

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INTRODUCTION.

In the application of rainfall records to any purpose it is always desirable and often indispensable that the record should be complete. Interpolation of rainfall may be required for a variety of purposes:

1. To fill in a missing record for one or more months.
2. To fill in records for one or more years.
3. For the determination of the rainfall in a single shower or for a certain day or for a particular storm.
4. To find the rainfall at a given location where no record has been kept, either the mean being required or the rainfall for a given period, storm, day, or shower.

In general the accuracy of the result increases with the length of the period for which interpolation is made. The rainfall for a year can usually be interpolated with a smaller percentage of error than rainfall for a month and this, in turn, can be interpolated more accurately than the rainfall for a storm, day, or shower. In the adoption of methods for interpolation of missing rainfall values the labor involved as well as the accuracy attainable must be considered. Methods of interpolation which are simple and which give excellent results when applied to the determination of missing annual or monthly values may and usually will not be equally well adapted to the interpolation of missing values for a given storm, day, or shower.

The determination of the rainfall amount at a given place in a given storm, day, or shower forms a separate problem, especially in case where no records whatever have been kept at the location in question. The present discussion is confined to the problem of determining missing rainfall values at locations where some records exist, either antecedent or subsequent to the missing interval, or both. It is in the form of monthly results that rainfall data are most often published and used and the completion of annual records often involves supplying data for missing months only. Anyone having occasion to use the rainfall records in a given locality will do well to make the necessary interpolations in a careful and reliable manner in the first instance of their application, thus rendering the records available in complete form for future use without further labor.

This discussion is confined to interpolation at the location of an existing rain-gage station. In all such cases there are some records at the location for which data are required, which may serve as guides to interpolation for the missing intervals.

Missing months within the body of a rainfall record are the result of three principal causes:

1. Absence or illness of observer.
2. Accidents to the rain gage or record.
3. Changing of observers.

Methods of interpolating missing records may be classified as follows:

1. Those dependent on records at the same station only.
a. The normal method.
b. Mean of preceding and following months.
c. Mean of the same month in preceding and following years.
d. Angot's method.
2. Methods depending on contemporaneous records at surrounding stations alone.
a. Substitution of the record for the nearest station.
b. Mean of three surrounding stations.
c. Inclined plane method.
3. Methods utilizing data for both the station of interpolation and for surrounding stations.
a. Fournie method.
b. Fournie-Horton method.
c. Abnormality method.
d. Angot-Horton method.
e. Angot-Leach method.

Some of these methods make use of contemporaneous records only, i. e., those at surrounding stations for the month or year to be interpolated, or those for the next preceding and following months, or years. Contemporaneous methods include:

1–*b*. Mean of preceding and following months.
1–*c*. Mean of same month in preceding and following years.
2–*a*. Nearest station method.
2–*b*. Mean of three surrounding stations.
2–*c*. Inclined plane method.
3–*d*. Correction ratio method.

Other methods require the use of monthly or annual normals or long term means at one or more stations. These include:

1–*a*. The normal method.
1–*d*. Angot method.
3–*a*. The Fournie method.
3–*b*. Fournie-Horton method.
3–*c*. The abnormality method.
3–*d*. Angot-Horton method.
3–*e*. Angot-Leach method.

CORRELATION AT ADJACENT STATIONS.

It would be expected that the accuracy obtainable in the use of the precipitation at one station for the determination of the precipitation at an adjacent station would depend to some extent on the degree of correlation between recorded rainfall amounts at the two stations. Selecting a group of stations in California with marked seasonal rainfall so as to eliminate uncertainties at the end of the hydrologic year, the coefficients of correlation of the seasonal total rainfall between adjacent stations were found as follows:

Correlation of seasonal total rainfall between adjacent stations.

	Years, incl.	K_e
North Bloomfield and—		
Grass Valley, Calif.	1873–74 to 1885–86 1895–96 to 1903–04	0.808
Iowa Hill, Calif.	1879–80 to 1885–86 1895–96 to 1908–09	0.804
Bowman Dam, Calif.	1899–1900 to 1908–09	0.620
Blue Canyon, Calif.	1889–90 to 1908–09	0.770
Nevada City, Calif.	1889–90 to 1908–09	0.930
Towle, Calif.	1895–96 to 1901–02	0.671
Truckee, Calif.	1871–72 to 1885–86 1895–96 to 1908–09	0.626
Cisco, Calif.	1871–72 to 1885–86 1895–96 to 1908–09	0.716
Cisco, Calif.	1900–01 to 1908–09	0.645

The location of the stations are shown on Figure 1. The coefficients are relatively high.

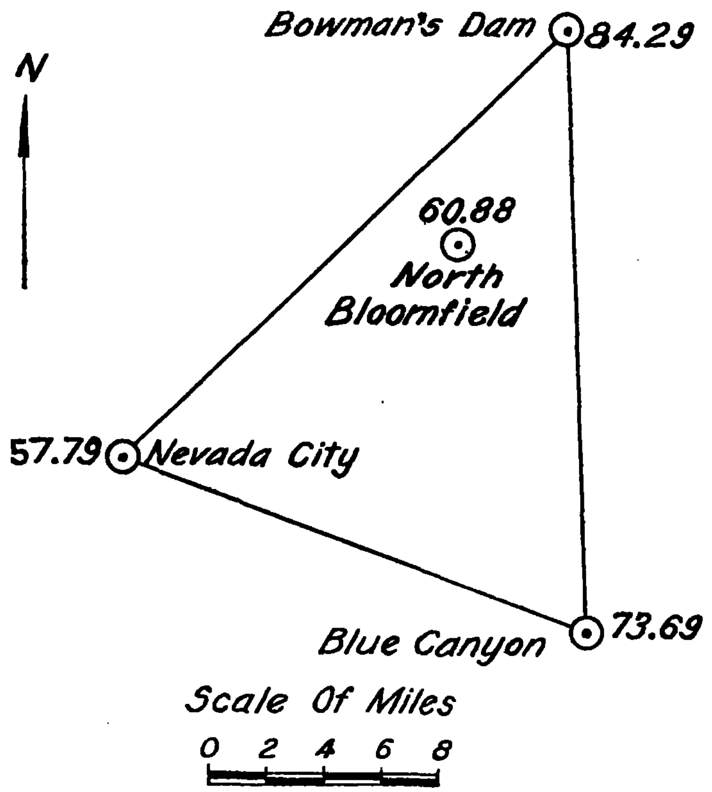

FIG. 1.—Location of rainfall stations, California group. (Figure gives mean annual rainfall.)

The correlation between monthly rainfall amounts may, however, be different from that between the seasonal totals. Monthly coefficients were worked out for 12 months selected at random, one month from each of 12 years for 6 pairs of stations, the locations being shown in Figures 1 and 2.

Correlation coefficients for adjacent stations for 12 calendar months selected at random.

North Bloomfield, Calif., and Bowman's Dam, Calif.	0.98
North Bloomfield, Calif., and Blue Canyon, Calif.	0.93
North Bloomfield, Calif., and Nevada City, Calif.	0.994
New Iberia, La., and Lafayette, La.	0.744
New Iberia, La., and Abbeville, La.	0.75
New Iberia, La., and Franklin, La.	0.87

CORRELATION OF RAINFALL IN CONSECUTIVE MONTHS OR YEARS.

In view of the convenience of using the record for preceding and subsequent intervals at the interpolation station as a basis for rainfall interpolation, it is of interest to determine the extent of correlation between rainfall amounts in a given month or year and in the corresponding intervals for preceding and subsequent years. The rainfall for the month of April at Albany, N. Y., is shown graphically on Figure 3 in comparison with the mean precipitation for the preceding and following months. The calculated correlation coefficients between the rainfall on a given month at Albany and the mean of the preceding and following months are as follows:

Correlation of coefficients between a given month and the mean of the preceding and following months.

(Albany, N. Y., 1874–1915, 42 years.)

April	−0.0539
July	+0.1004
November	+0.194

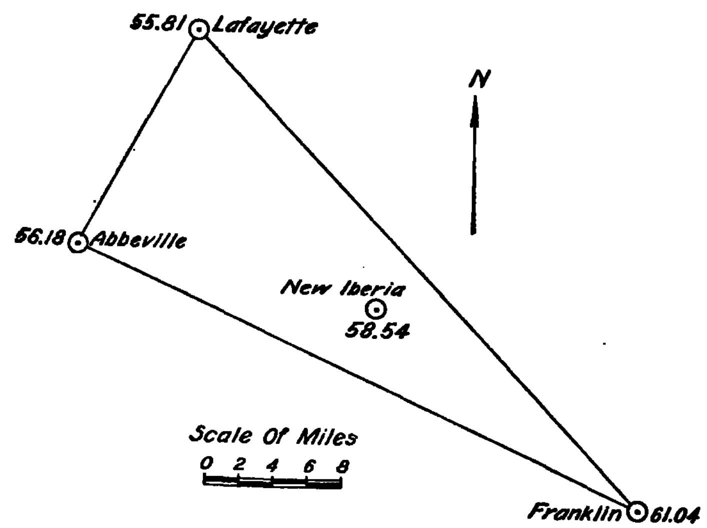

FIG. 2.—Location of rainfall stations, Louisiana group. (Figure gives mean annual rainfall.)

The coefficients are lower than in the case of correlation between simultaneous intervals at adjacent stations.

Hessling [1] found the correlation between different pairs of months as to rainfall and temperature, respectively, for 24 stations in the corn region of Argentina, as follows:

In some cases there is a fair degree of correlation for rainfall but in general it is less than for temperature.

Months correlated.	K_e Precipitation.	K_e Temperature.
October–November	0.22	0.63
October–December	0.48	0.18
October–January	0.07	0.53
November–December	0.29	0.56
November–January	0.03	0.51
December–January	0.33	0.50

[1] Relation between the rainfall, the temperature, and the yield of corn in Argentina. MO. WEATHER REV., Oct., 1921, 49:545.

Peck and Snow [2] have determined the correlation between the rainfall of each month of the year in England

[2] The correlation of rainfall, *Quar. Jour. Roy. Met. Soc.*, Oct. 1913, pp. 307–316.

and that of the remaining 11 months, for each of four years, 1908 to 1911, with the following results:

Coefficients of correlation of the rainfall of each month with that of the remaining months of the year.

(Mean for four years 1908-1911.)

January	+0.31	July	0.00
February	+0.15	August	+0.15
March	+0.29	September	+0.12
April	+0.20	October	+0.25
May	+0.15	November	+0.19
June	+0.04	December	+0.25

Here again the coefficients are relatively low; in fact, there is no appreciable correlation between the rainfall of the summer months in England and that of the remainder of the year. This is probably the effect of thunderstorms in these months, whereas cyclonic and orographic rain predominates in the remaining months of the year. The correlation coefficients between rainfall in a given year in England and that in the preceding and following years have also been determined by Peck and Snow. Here the resulting coefficients are relatively much larger than those obtained for single months compared with the preceding and following months. This indicates that the use of the measured precipitation for the preceding and following years at a given station may be much more reliable as a means of interpolating annual than monthly rainfall.

Correlation coefficients showing the relation of rainfall in a given year to that in the preceding and following year.

	1908	1909	1910	1911
1908	+1.00	+0.69	+0.70	+0.57
1909	+0.69	+1.00	+0.75	+0.64
1910	+0.70	+0.75	+1.00	+0.71
1911	+0.57	+0.64	+0.71	+1.00

From a study of long term rainfall records at Greenwich, Glasgow, Greenock and Dundee, Russell[3] found that the coefficients of correlation between rainfall amounts in successive months were always below 0.50.

The average correlation coefficient for the 48 pairs of monthly cases at the four stations is near zero. There were 17 cases of negative and 31 cases of zero or plus coefficients. For the pairs of winter months, November and December to January and February, inclusive, there are consistent positive correlations following an average value of about 0.25. In general the results indicate that there is but little positive correlation between rainfall amounts for two successive years at the same station, especially during the summer season.

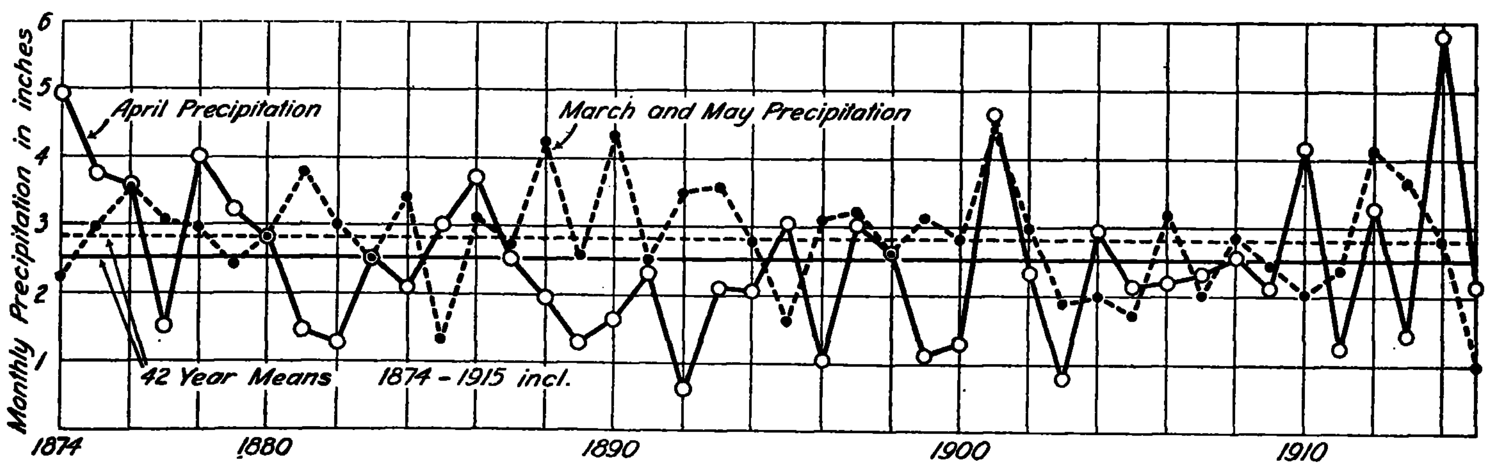

FIG. 3.—Relation between April rainfall and mean of preceding and following months. Albany, N. Y., 1874-1912, inclusive.

MONTHLY RAINFALL INTERPOLATION.

There are two principal conditions under which the interpolation of monthly rainfalls may be required:

1. To fill in gaps within the record at a given station, the previous and subsequent records both being available.
2. Extrapolation to extend a record so as to make it complete for a chosen period.

Both these cases are here considered under the general term "interpolation." In general (with one exception), methods applicable to the first case are also applicable in the case of extrapolation, and the accuracy obtainable in the two cases is usually about the same. There are often some months missing from otherwise excellent rainfall records. Obviously a record for 20 years, containing say 10 missing months scattered through 5 different years, is better if completed than if only the 15 complete years are utilized. The record when completed represents actual observations for 19 years and 2 months, and even should there happen to be considerable error in the interpolation of the remaining 10 months, the resulting mean for 20 years is likely to be nearer to the true long term mean than is the mean for the 15 complete years only. It is not infrequently the case that one or more months are missing from almost every year, even where such a fragmentary record is the only one available, and the choice lies between discarding the record altogether, or in some way completing it.

Relation curves between the rainfall amounts at adjacent stations can be derived if fairly long records are available for both stations. The accuracy of most interpolation methods depends on the relation between the precipitation at adjacent stations.

Having given the monthly relation curve between two stations and the precipitation at the base station, corresponding precipitation shown by the relation curve gives a value of the quantity to be interpolated. If relation curves are available for three base stations the mean of the three resulting interpolated values may be used. This is perhaps the most rational of all methods of rainfall interpolation, especially if three stations are used and the three results are given weights dependent on the relative distances of the base stations from the interpola-

[3] *Quar. Jour. Roy. Met. Soc.*, July, 1922, p. 225.

tion station. If interpolations are required in different months of the year, then the use of this method involves the derivation of a rainfall relation curve for each month for which interpolations are to be made, and if three base stations are used three sets of curves are necessary. The labor of using the method for monthly values is so excessive that it has not been given further consider ation.

In case of the interpolation of annual rainfall amounts only one relation curve is required between the interpolation station and each of the base stations. The method is therefore much better adapted to interpolation of annual than of monthly rainfall.

In discussing multiple station methods the use of three surrounding stations is mainly considered, this being the most usual case. Instances will, however, arise where only one or two adjacent station records are available and others where there are four equally applicable. Adaptation of the methods described to these special conditions will be readily perceived. If there is choice of stations, those having the highest correlation with the station for which interpolation is to be made should be selected.

The station for which interpolation is required is designated the "interpolation station," whereas the surrounding stations for which records are available for the period in question and which are utilized in the interpolation are designated "base stations."

Obviously, methods involving the use of normal monthly or annual rainfall are inapplicable where the available records are of short duration.

Notation.—The following notation is used in describing the different methods of interpolation and weighting of the results.

d = rainfall at interpolation station.

a, b, c = rainfall at surrounding stations.

A, B, C, D = normal annual rainfall at the various stations.

D_n = normal rainfall for the same month at the interpolation station.

Subscripts 1, 2, relate to values for preceding and following months.

Subscripts p, f, relate to values for the same month in the preceding and following years.

X_a, X_b, X_c = distances of base stations from the interpolation station.

W_a, W_b, W_c = relative weights of results derived from stations a, b, and c.

Characteristics of the different methods are as follows:

1–*a*. *Normal method.*—This consists in substituting the mean rainfall for a given month as determined from the longest available record at the interpolation station. Obviously the normal method can only be used where the station has been maintained long enough to give fairly good normals for the different months. In applying this method no effort is made to take into account the abnormality of the rainfall for the month to be interpolated. At the same time, if a large number of months are missing from a record, substitution of the normal values in place of the actual, which are unknown, will give theoretically the same mean for the whole record as if the actual values had been utilized. There is, however, no reason for belief that the rainfall for a particular month agrees in any close degree with the normal for that month, and since some information may generally be obtained as to the abnormality of the precipitation in any particular month, this method must be considered as not conforming to the requirement of making the best use of the available information.

As regards annual results the error in the mean resulting from the substitution of the normal for the actual precipitation in any one year decreases as the length of record increases.

If r is the ratio of the actual rainfall in the given year to the mean,

$$\frac{\text{Apparent mean, } N \text{ years}}{\text{true mean}} = \frac{N-1+r}{N}$$

This becomes unity if the ratio r is unity.

1–*b*. *Mean of preceding and following months.*—If the precipitation in a given month is abnormally high, the mean of the preceding and following months is rather likely to be high, and vice versa. Again, if the precipitation varied uniformly from month to month, the mean for any month would be equal to the mean of the preceding and following months. The precipitation does not, however, vary in a uniform manner, and the mean of the preceding and following months will generally be less than the true precipitation for the maximum month of the year or season and too large for the minimum month of the year or season. This method has the advantage that it is based solely on records at the station of interpolation, and, furthermore, requires only the use of the contemporaneous three months. No long term means are needed. It is, therefore, exceedingly simple to apply and can be used as well for a very short as for a long record, but can not be used for extrapolation.

1–*c*. *Mean of the same month in the preceding and following years.*—It is found by statistical studies that years with rainfall either above or below the mean tend to run in groups in an irregular periodicity. If the distribution of years of high or low rainfall, singly or in groups, was a matter of chance, then the probable number of groups of n-like years (all above or all below the mean) in 100 years' record would be:

n=	1	2	3	4	5	6	7	8	9	10
	50	25	12.5	6.25	3.12	1.56	0.78	0.39	0.195	0.0925

For any year excepting the maximum or minimum year of such a group or period of like years, the precipitation in a given month is likely to be approximately the mean of that for the same month in the preceding and following years. This method is subject to the same errors and limitations as to the use of the mean of the preceding and following month, and it can not be used for extrapolation. In general it gives better results when the missing months fall in a group of several like years than when they fall in an isolated year. In the latter case if the true value for the missing month is high, the corresponding months in both preceding and following years are likely to be low and vice versa. For this condition the mean of the same months in preceding and following years may be seriously in error.

1–*d*. *Angot method.*—Angot developed what are known as pluviometric coefficients; these are essentially the ratios of the precipitation amounts in the different months to the yearly total. These in general are more nearly constant for a given month than is the actual monthly precipitation. This would be expected since, for example, an excessive precipitation in a given month adds to the yearly total and vice versa, in both cases resulting in a tendency to maintain constancy in the pluviometric coefficient.

Similarly the pluviometric coefficients at adjacent stations are generally more nearly equal than are the actual rainfall amounts. The use of pluviometric coefficients for the base or for surrounding stations should apparently afford a reliable method of interpolation. Unfortunately, the true Angot pluviometric coefficient can not be determined at the interpolation station for the year in which interpolation is to be made, since the record is wanting for at least one month. A modified coefficient can be used. Expressing the normal ratio of precipitation in the missing month to the total normal precipitation in the remaining 11 months by C'', then if $\Sigma P''$ is the total precipitation in the remaining 11 months of the given year, the precipitation for the missing month could be estimated by the formula,

$$P = C''\Sigma P''$$

This method would, however, be very laborious. It has accordingly been modified, probably at some sacrifice of accuracy, by using instead of C'' the ratio

$$C = \frac{D}{D_1 + D_2}$$

or the ratio of the normal for the missing month to the mean of the normals for the preceding and following months; then

$$d = C\frac{d_1 + d_2}{2}$$

This method is similar to the use of the mean of the preceding and following months but a correction is

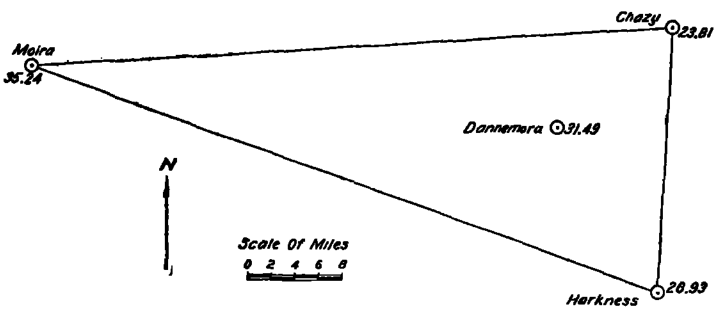

FIG. 4.—Location of rainfall stations, New York group. (Figure gives mean annual rainfall.)

made for the unequal rate of variation of rainfall from month to month. The method is wholly dependent upon records at the station of interpolation. It is less laborious than methods utilizing long term means for surrounding stations, but is in general more laborious than any of the methods described dependent on contemporaneous records alone, although if a large number of values is to be interpolated, the work by either this or the other methods involving long term means is proportionally decreased as compared with the case where only a small number of interpolations is required, since the monthly means once determined answer for all the interpolations. Frequently the monthly means are available at the outset. This method can, of course, be applied to three surrounding stations, but it then becomes more laborious than Fournie's method without any apparent advantages.

2–*a*. *Nearest station.*—The substitution of the record at the nearest adjacent station for a missing monthly record is not an uncommon procedure. It is perhaps the simplest of all methods of obtaining a value to fill out a missing month. There is generally a fairly good correlation between monthly precipitation at adjacent stations. The correlation, however, might be perfect and yet the values for one station be widely different from those for the other, owing to a constant difference which does not appear in the correlation coefficient. This source of error is eliminated by the use of Angot's pluviometric coefficients (method 2–*d*). Where the means for the two stations are substantially the same, direct substitution of the value for the nearest station frequently gives good results for closely adjacent stations. If, however, there is no single closely adjacent station, but if there are several nearly equidistant but more remote stations, the use of the precipitation at the nearest station alone is not justified. The method can be applied either to interpolation or extrapolation, and since it does not require the determination of a mean, it can be applied to a short as well as to a long record.

2–*b*. *Mean of three surrounding stations.*—This method should theoretically have a much greater accuracy than the use of the nearest station alone. Furthermore, since surrounding stations on different sides of the interpolation station are to be used, the effect of local storms which may occur at one station but not at another is more likely to be taken into account. It involves but little labor and has all the other advantages of the use of the nearest station record.

2–*c*. *Inclined-plane method.*—This method was devised by the author with a view to applying simultaneous or or contemporaneous records in the most logical manner possible, thus obtaining the best practicable result with

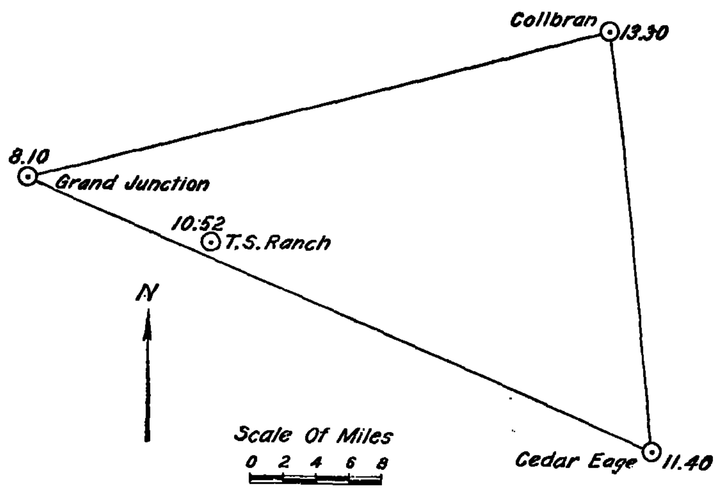

FIG. 5.—Location of rainfall stations, Colorado group. (Figure gives mean annual rainfall.)

the least expenditure of labor, since all methods dependent on simultaneous or contemporaneous records alone are much simpler of application than methods involving the use of long-term means. It depends upon the principle that the position of a plane is completely determined by the coordinates of three points in the plane. In the absence of information to the contrary, the best assumption which can be made is that rainfall varies uniformly between adjacent stations.

Select three base stations, A, B, C, Figure 6, surrounding the interpolation station. On a suitable map showing the relative positions of the stations, connect any pair, A, B, of the base stations by a line, and erect perpendiculars to this line at the two stations, and measure off on each perpendicular a length proportional to the precipitation at that station for the period to be interpolated. On the assumption of uniform variation, the precipitation at any point along this will be proportional to the ordinate from the base line to the line connecting the two plotted points. Draw another line from the third base station, C, through the interpolation station, D, intersecting the base line AB at E. Erect a perpendicular to AB at E, intersecting the line FG at H. The precipitation at E is assumed to be proportional to EH. Draw perpendiculars to CE, one at C proportional to the precipitation at that station, and one at B equal to EH. Connect these by a line JK. Then a perpendicular DL at the interpolation station will have a length proportional to the precipitation at D. The graphical construction is extremely simple. Only simultaneous records are used and the method involves but little more labor than the use of the mean of the three surrounding stations, but it is more logical. The direct use of the mean of three surrounding stations gives equal weight to each of the stations, although they may be at widely different distances from the interpolation station. The weight given to remote stations in the inclined-plane method decreases as the distances increases.

The inclined plane method of combining the results of data for surrounding stations can also be used in conjunction with the Fournie and other multiple station methods. It is to be considered, therefore, as a principle rather than as being restricted to the narrow limits of a method of interpolation.

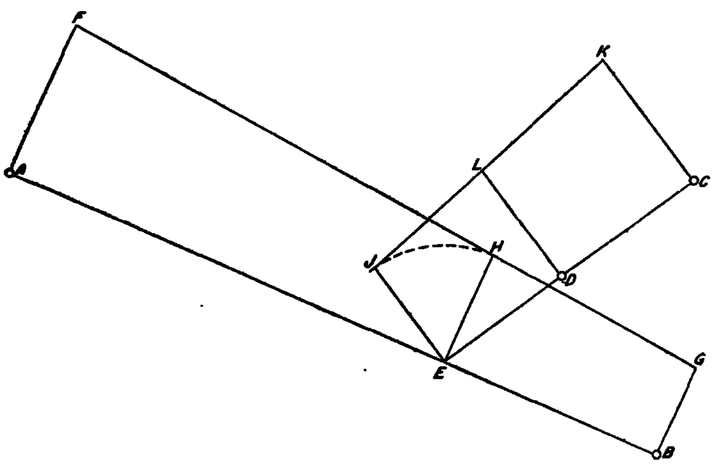

FIG. 6.—The inclined-plane method.

It can readily be shown from the geometrical construction of Figure 6 that—

$$P_d = K_1\, P_a + K_2\, P_b + K_3\, P_c$$

where

$$d_1 = \frac{AE}{AB},\ d_2 = \frac{ED}{EC},\ \text{Figure 6},$$

and

$$K_1 = 1 - d_1 - d_2 + d_1\, d_2$$

$$K_2 = d_1 - d_1\, d_2,\ K_3 = d_2;$$

also

$$K_1 + K_2 + K_3 = 1$$

or the sums of the coefficients is unity, providing an easy check on computations.

3–*a*. *Fournie method.*—This method has been extensively used for interpolation of missing rainfall years. It is equally applicable to the interpolation of missing months, but like all other methods, gives less accurate results for monthly than for annual interpolations, owing to the relatively greater variability of rainfall for short periods than for full years. To apply this method, three surrounding stations are selected for each of which there is a rainfall record for a period of several years simultaneous with the rainfall record at the interpolation station. Calling means for these three stations A, B, and C, and calling the mean for the interpolation station D, the ratios $\frac{D}{A}$, $\frac{D}{B}$, and $\frac{D}{C}$ are worked out for the simultaneous periods covered by all four records. Calling the actual precipitations at the three base stations for the period for which interpolation is required a, b, and c, respectively, these values are multiplied by the corresponding ratios and the mean of the three products taken as the interpolated rainfall for the interpolation station. The method is laborious, especially in view of the fact that in many cases gaps will be found in the records for the base stations which must themselves be filled out before long-term ratios can be computed. Sometimes one of the base stations chosen is necessarily much more remote from the interpolation station than are the others. Fournie's method gives equal weight in the result to the values obtained from the different base stations, whereas it is likely that the precipitation for the given month at the interpolation station conforms more closely to that at a nearby station than to that at a remote station. Obviously the Fournie method can only be applied where records have been kept for a number of years. It does not depend on contemporaneous records and considerable research is often necessary to compile the data for computing the ratios of the means, even where the records for the base stations are complete.

3–*b*. *Fournie-Horton method.*—The Fournie method gives equal weight to three surrounding stations, but utilizes data at the interpolation station. The inclined-plane method does not utilize data for the interpolation station. A combination of the inclined-plane and Fournie methods serves to make use of the data for the base station and at the same time provides for weighting the results obtained at surrounding stations. The combination of the two methods has been accomplished by computing the ratios of the normal precipitation at the interpolation station to the normal at each of the three base stations, as in the Fournie method. The values of these three ratios are plotted by the inclined plane method and a correction factor obtained, which is applied to the precipitation determined by the inclined plane method from simultaneous records at three surrounding stations. This involves two applications of the inclined-plane method for each calculated interpolation. An adaptation of this method, simpler and apparently equally good, consists in first determining the three values of d from the Fournie ratios $\frac{D}{A}a$, $\frac{D}{B}b$, $\frac{D}{C}c$ and then applying the inclined plane method to these values to determine the interpolation value.

3–*c*. *The abnormality method.*—The abnormality method is based on the departure of the precipitation at adjacent stations from the normal or mean precipitation for the month to be interpolated. The normal precipitation for the given month is first determined for the base stations and the ratio between the actual precipitation and the mean is then found for each of these stations. This ratio indicates the departure from the mean, or the abnormality for the month to be interpolated. The precipitation at the interpolation station is found by multiplying the normal rainfall for the same month at this station by the direct or weighted mean of the abnormality ratios for the base stations.

The disadvantages of this method lie in the necessity of having long-time records at adjacent stations and in the labor involved in computing the means.[4]

[4] This method in reality is identical with the Fournie method, the only difference being in the order in which the computations are made.

3–d. Correction-ratio method.—This method was devised in order to utilize the data for surrounding stations and at the same time give the data for the base station the greatest possible weight, and yet base the interpolation wholly on contemporaneous records so as to avoid the labor of methods dependent on long-term means. It consists in computing the ratios of the actual rainfall at each base station for the month to be interpolated to the sum of the rainfall amounts for the preceding and following months at the same stations, giving three coefficients similar to the single coefficient used in the Angot method. The factors so obtained take into account the abnormality of the precipitation for the month to be interpolated, both as a result of non-linear variation in the rainfall at the interpolation station and also as a result of any local abnormalities of this particular month. The direct or weighted mean of these coefficients is then applied to the sum of the precipitation for the preceding and following months at the interpolation station.

Expressed symbolically,

$$C_a=\frac{a}{a_1+a_2},\quad C_b=\frac{b}{b_1+b_2},\quad C_c=\frac{c}{c_1+c_2}$$

Then if these coefficients have weights $w_a+w_b+w_c=1$

$$P=(C_a\ W_a+C_b\ W_b+C_c\ W_c)\ (d_1+d_2)$$

3–e. Horton method.—This method consists in computing the ratios of the precipitation for the month to be interpolated to the sum of the precipitations in the preceding and following months at each of the three surrounding stations precisely as in the correction-ratio method. A weighted mean ratio is then obtained by applying the inclined-plane method to these three values, and the interpolated value equals the product of the weighted correction ratio multiplied by the sum of the precipitation amounts for the preceding and following months at the interpolation station.

3–f. Leach method.—This is the same as the preceding except that the weights given in the three correction ratios are taken in inverse proportion to the relative distances of the base stations from the interpolation station. It will be noted that the last three methods described all depend in part on the correlation between rainfall amounts in successive months at a given station, and this correlation, as already shown, is frequently small. These methods, however, depend in a much larger degree on the correlation between rainfall amounts at adjacent stations in the corresponding months, and this is usually fairly large.

METHODS OF WEIGHTING INTERPOLATED VALUES.

In applying the methods of interpolation where several surrounding stations are used, each station yields in general a value of the interpolated quantity. These may be given equal weights, in which case the adopted value of the interpolated quantity is the arithmetic mean of the several (usually three) values, or the individual values may be given weights, depending on the locations of the stations or their similarity. In general when three base stations are used, if W_a, W_b, and W_c are the weights assigned to the interpolated values, these weights being chosen relative to such a scale that $W_a+W_b+W_c=1$, then $d=W_a\ d_a+W_b\ d_b+W_c\ d_c$.

These weights may be arrived at by several methods—

(*a*) By judgment.

(*b*) Inversely as the relative distances of the base stations from the interpolation station.

(*c*) The inclined-plane method.

Weighting by judgment.—Theoretically this is perhaps the best method if properly applied, since it is possible in using it to take into account not only the relative positions of the stations, but the nature, if known, of the rainfall variation between them, and the differences in rainfall causes applying to each.

Consider, for example, an interpolation station on the plains near the foot of a mountain range, with three interpolation stations, two on the plains and one on the mountains. Now suppose the conditions are such that a large proportion of the rainfall on the plains is convective, while at the mountain station the rainfall is more largely orographic. Obviously the plains stations should be given greater weight than the mountain station, if all were equidistant from the interpolation station. Among disadvantages of weighting by judgment are—

(1) The factors affecting the proper weights to be applied are, except relative distances, in general unknown, or not quantitatively known; therefore,

(2) Different operators using the same data will not obtain the same results.

Weighting by inverse distances.—If X_a, X_b, X_c, are the distances of the base stations from the interpolation station, in any linear units, then if weights are assigned to the three stations each inversely proportional to its distance from the interpolation station and on such a scale that

$$W_a+W_b+W_c=1$$

the numerical values of the weights can be derived as follows: Take reciprocals of X_a, X_b, and X_c. Let

$$\frac{1}{X_a}+\frac{1}{X_b}+\frac{1}{X_c}=M$$

then

$$W_a=\frac{1}{M\ X_a},\quad W_b=\frac{1}{M\ X_b},\quad W_c=\frac{1}{M\ X_c}.$$

These relative weights once determined can be applied to all interpolations involving a given group of stations.

SUMMARY OF INTERPOLATION FORMULAS.

For convenience reference the various methods are summarized in analytical form as follows:

(1–*a*) Normal method, $d=d_n$.

(1–*b*) Mean of preceding and following months—

$$d=\frac{d_1+d_2}{2}$$

(1–*c*) Mean of same month in preceding and following years—

$$d=\frac{d_p+d_f}{2}$$

(2–*a*) Nearest station, $d=a$, b or c, as the case may be.

(2–*b*) Three surrounding stations. For equal weights,

$$d=\frac{a+b+c}{3}$$

or in general—

$$d_a=a,\ d_b=b,\ d_c=c$$

(3–*a*) Fournie's method—

$$\text{Let } r_a=\frac{D}{A},\ r_b=\frac{D}{B},\ r_c=\frac{D}{C}$$

For equal weights—

$$d=(r_a a+r_b b+r_c c)/3$$

or in general—

$$d_a=r_a a,\ d_b=r_b b,\ d_c=r_c c$$

(1–*d*) Angot method.

$$d=\frac{D}{D_1+D_2}(d_1+d_2)$$

(3–*c*) Abnormality method, equal weights.

$$d_a=\frac{a}{A}D,\ d_b=\frac{b}{B}D,\ d_c=\frac{c}{C}D$$

(3–*d*) Correction ratio method.

$$C_a=\frac{a}{a_1+a_2},\ C_b=\frac{b}{b_1+b_2},\ C_c=\frac{c}{c_1+c_2}$$

$$d=(C_a+C_b+C_c)\quad(d_1+d_2)$$

or in general—

$$d_a=\frac{C_a}{d_1+d_2},\ d_b=\frac{C_b}{d_1+d_2},\ d_c=\frac{C_c}{c_1+c_2}$$

METHODS OF WEIGHTING.

For three stations with weights $w_a+w_b-w_c=1$.

$$d=d_a\,w_a+d_b\,w_b+d_c\,w_c$$

Weights by inverse distance ratios.

$$w_a=\frac{1}{M\,X_a},\ w_b=\frac{1}{M\,X_b},\ w_c=\frac{1}{M\,X_c}$$

where

$$M=\frac{1}{X_a}+\frac{1}{X_b}+\frac{1}{X_c}$$

Weights by inclined plane method.

$$W_a=K_1=1\ -d_1-d_2+d_1\,d_2$$
$$W_b=K_2=d_1-d_1\,d_2$$
$$W_c=K_3=d_2$$

EXAMPLES OF MONTHLY RAINFALL INTERPOLATION.

In order to compare the different methods, the labor involved in applying them, and their relative values and accuracy, a series of examples was first chosen to which each of the methods described has been applied. These examples were selected to represent four different regions with conditions covering as nearly as possible the range of variation in amount and distribution of rainfall in the United States. The locations of the four groups of stations are shown on Figures 1, 2, 4 and 5.

1. Eastern interior type: Represented by Dannemora, N. Y. Moderate rainfall, quite uniformly distributed throughout the year; variability low.

2. Tropical type: Heavy rainfall, quite uniformly distributed throughout the year but with medium variability, represented by New Iberia, La.

3. Arid type: Very light precipitation, somewhat irregularly distributed; high variability. Represented by T. S. Ranch, Colo.

4. Concentrated seasonal precipitation, or monsoon type: Heavy precipitation during winter months (mostly snow), little or none during certain summer months; high variability. Represented by North Bloomfield, Calif.

The stations chosen include extreme conditions as to snowfall amount and rainfall variability and are probably susceptible to less accurate interpolation of records than the average for central and eastern United States.

In order to test the different methods, a series of 12 months was selected from the record for each station referred to, the selection being made at random, but so that the months chosen for interpolation were not consecutive. Care was taken to secure stations for interpolation such that the complete records were available for the period covered by the interpolations at each of three nearby surrounding stations.

The results obtained by the 12 methods of interpolation used are given in the accompanying Table 1. The first column for each method shows the 12 interpolated values. The second column for each method gives the actual errors in inches. Footings of the columns give the average arithmetic error of the monthly interpolations and the total algebraic error of the 12 interpolations for each station and method.

The comparative results by different methods for each interpolation station are summarized in Table 2. The first section shows the average arithmetic error per month in inches, and the second section the average algebraic error per month in inches.

The average results in inches and also percentages of the true monthly precipitation are further summarized in Table 3. Methods using contemporary records at surrounding stations give much smaller arithmetic error than the simpler methods using data for the interpolation station only, but there is not much difference between the two groups of methods as regards algebraic error, since most methods in both groups involve constant errors due to differences between the means at the interpolation station and the base stations.

TABLE 1.—*Comparison of methods of interpolating missing monthly precipitation records.*

Month and year.	Actual precipitation, inches.	1-a. Normal method.		1-b. Mean of preceding and following months.		1-c. Mean of same month preceding and following year.		1-d. Angot method.		2-a. Nearest station.		2-b. Mean of three surrounding stations.		2-c. Inclined plane.		3-a. Fournie method.		3-b. Fournie-Horton method.		3-c. Abnormality method.		3-e. Horton method.		3-f. Leach ratio.	
		Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.	Value.	Error.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
Interpolated precipitation at Dannemora, N. Y., from Chazy, Harkness, and Moria records, 1906-1915.																									
Jan., 1907	1.26	2.16	0.90	1.81	0.55	1.63	0.37	1.59	0.33	1.25	−0.01	1.47	0.21	1.30	0.04	1.58	0.32	1.79	0.53	1.83	0.57	1.77	0.51	1.76	0.50
Feb., 1910	3.88	2.15	−1.73	1.51	−2.37	2.74	−1.14	1.49	−2.39	2.20	−1.68	3.25	−.63	2.90	−.88	3.49	−.39	4.00	.12	3.62	−.26	5.40	1.62	5.44	1.56
Mar., 1909	2.30	2.20	−.10	2.95	.65	3.75	1.45	2.92	.62	.98	−1.32	1.63	−.67	1.50	−.80	1.77	−.53	2.07	−.23	1.83	−.47	1.62	−.68	1.59	−.71
Apr., 1911	0.74	2.28	1.54	2.74	2.00	2.28	1.54	2.69	1.95	.33	−.41	0.66	−.12	.54	−.20	.68	−.06	.74	0	.91	.17	.88	.14	.86	.12
May, 1913	2.24	3.46	1.22	2.32	.08	3.08	.84	3.16	.92	2.36	.12	2.35	.11	2.72	.48	2.63	.39	3.75	1.31	2.76	.52	4.06	1.72	4.11	1.87
June, 1912	1.42	2.80	1.38	4.50	3.18	2.95	1.53	5.45	4.03	1.50	.08	1.25	−.17	.90	−.42	1.36	−.06	1.24	−.18	1.37	−.05	.95	−.47	.92	−.50
July, 1910	3.04	3.17	.13	2.56	−.48	2.87	−.17	2.92	−.12	1.95	−1.09	2.87	−.17	2.70	−.34	1.14	−1.90	3.72	.68	3.11	.07	2.46	−.58	2.48	−.56
Aug., 1912	4.58	2.74	−1.84	4.58	0	2.28	−2.30	3.84	−.74	4.69	.11	3.92	−.66	4.05	−.53	4.49	−.09	5.58	1.00	3.81	−.77	3.63	−.95	3.78	−.80
Sept., 1907	6.93	3.37	−3.56	1.33	−5.60	2.03	−4.90	3.39	−3.54	6.50	−.43	6.04	−.89	6.27	−.66	6.92	−.01	8.65	1.72	7.05	.12	4.66	−2.27	4.69	−2.24
Oct., 1908	1.66	2.53	.87	1.42	−.24	1.45	−.21	1.38	−.28	.93	−.73	1.29	−.37	1.20	−.46	1.41	−.25	1.65	−.01	1.39	−.27	1.62	−.04	1.59	−.07
Nov., 1913	2.36	1.83	.53	3.42	1.06	2.88	.52	2.36	0	.22	−2.14	1.38	−.98	1.06	−1.30	1.38	−.98	1.46	−.90	1.17	−1.19	2.44	.08	2.38	.02
Dec., 1914	2.14	2.74	.60	2.00	−.14	2.76	.62	2.74	.60	.80	−1.34	1.95	−.19	1.60	−.54	2.00	−.14	2.20	.06	2.44	.30	2.10	−.04	2.11	−.03
Year	32.55	31.43	−1.12	32.46	−.09	30.72	−1.83	33.93	+1.38	23.71	−8.84	28.06	−4.49	26.74	−5.81	28.85	−3.70	36.85	+1.30	31.29	−1.26	31.59	+0.96	31.71	−.84
Mean			1.20		1.36		1.30		1.30		.79		.43		.55		.43		.56		.40		.76		.75
Interpolated precipitation at T. S. Ranch, Mesa Co., Colo., from Cedar Edge, Collbran and Grand Junction, 1892-1901.																									
Jan., 1899	0.29	0.68	0.39	0.42	0.13	0.38	0.09	0.39	0.10	0.42	0.13	0.54	0.25	0.39	0.10	0.52	0.23	0.43	0.14	0.45	0.16	0.44	0.15	0.38	0.09
Feb., 1893	1.01	0.73	−.28	.70	−.31	.92	−.09	.59	−.42	1.77	.76	2.69	1.68	2.11	1.10	2.54	1.53	2.34	1.33	1.80	.79	2.37	1.36	2.54	1.53
Mar., 1897	1.88	1.14	−.74	1.30	−.58	.68	−1.20	1.87	−.01	1.05	−.83	1.48	−.40	1.43	−.45	1.44	−.44	1.59	−.29	1.40	−.48	1.55	−.33	1.39	−.49
Apr., 1899	1.56	.84	−.72	.88	−.68	1.17	−.39	1.01	−.55	1.11	−.45	.79	−.77	.93	−.63	.83	−.73	1.03	−.53	.65	−.91	1.99	.43	1.72	.16
May, 1894	0.75	.79	.04	.37	−.38	.48	−.37	.43	−.32	.56	−.19	.59	−.16	.61	−.14	.60	−.15	.68	−.07	.50	−.25	1.63	.88	1.34	.59
June, 1898	0.15	.54	.39	1.34	1.19	1.20	1.15	.87	.72	.05	−.10	.28	.13	.13	−.02	.22	.07	.14	−.01	.19	.04	.17	.02	0.20	.05
July, 1895	1.73	.86	−.87	1.10	−.63	1.02	−.71	1.04	−.69	1.43	−.30	1.41	−.32	1.38	−.35	1.41	−.32	1.53	−.20	1.51	−.22	1.10	−.63	1.14	−.59
Aug., 1896	0.45	1.27	.82	3.00	2.55	1.17	.72	3.81	3.36	1.01	.56	1.10	.65	1.00	.55	1.08	.63	1.11	.66	1.34	.89	1.44	.99	1.39	.94
Sept., 1900	0.80	1.14	.34	.50	−.30	.34	−.46	.44	−.36	1.18	.38	1.50	.70	1.58	.78	1.51	.71	1.75	.95	1.51	.71	3.35	2.55	2.38	1.58
Oct., 1900	0.87	1.30	.53	.84	−.03	.05	−.82	1.33	.46	.14	−.73	.84	−.03	.87	0	.75	−.12	.97	.10	.98	.11	.31	−.56	.45	−.42
Nov., 1897	0.86	.50	−.36	1.21	.35	.40	−.26	.61	−.25	.33	−.53	1.65	.79	.58	−.28	.59	−.27	.64	−.22	.47	−.39	.39	−.47	.44	−.42
Dec., 1893	1.20	.72	−.48	.48	−.72	.96	−.24	.61	−.59	.50	−.70	1.28	.08	.76	−.54	1.15	−.05	.84	−.36	1.07	−.13	.52	−.68	.61	−.59
Year	11.55	10.52	−1.03	12.14	+0.59	9.44	−2.11	13.00	1.45	9.55	−2.00	14.13	2.58	11.77	+.22	12.64	1.09	13.05	1.50	11.87	+.32	15.26	3.71	13.98	+2.43
Mean			.50		.65		.54		.65		.47		.50		.41		.44		.41		.42		.75		.62
Interpolated precipitation at New Iberia, La., from Franklin, Lafayette, and Abbeville records, 1900-1909.																									
Jan., 1908	4.05	3.60	−0.45	3.36	−0.69	2.45	−1.60	5.14	1.09	[a] 3.29	−0.76	3.59	−0.46	3.60	−0.45	3.65	−0.40	3.44	−0.61	3.83	−0.22	2.96	−1.09	2.90	−1.15
Feb., 1901	6.05	4.91	−1.14	5.29	−0.76	5.40	−.65	4.81	−1.24	4.50	−1.55	5.10	−.95	5.50	−.55	5.48	−.57	5.26	−.79	4.76	−1.89	8.66	2.61	7.93	1.18
Mar., 1907	.80	3.84	3.04	3.86	3.06	1.82	−1.02	4.56	3.76	0.59	−.21	.61	−.19	.60	−.20	.62	−.18	.57	−.23	.60	−.20	.77	−.03	.74	−.06
Apr., 1902	4.30	4.95	0.65	4.35	.05	5.80	1.50	2.87	−1.43	3.51	−.79	4.19	−.11	4.05	−.25	3.88	−.42	3.87	−.43	4.47	.17	4.87	.57	4.89	.59
May, 1906	2.22	4.78	2.56	3.45	1.23	10.98	8.76	2.34	.12	1.45	−.77	1.88	−.34	2.30	−.08	1.87	−.35	2.20	−.02	1.71	−.41	2.48	.26	2.19	−.03
June, 1903	5.70	5.47	−0.23	3.65	−2.05	2.18	−3.52	5.44	−.26	4.22	−1.48	3.93	−1.77	4.05	−1.65	3.97	−1.73	3.87	−1.83	4.27	−1.43	5.95	.25	4.70	−1.00
July, 1905	10.58	9.33	−1.25	10.60	.02	10.60	.02	8.46	−2.42	8.67	−1.91	9.16	−1.42	9.70	−.88	9.24	−1.34	9.27	−1.31	8.25	−2.33	9.96	−.62	9.25	−1.33
Aug., 1904	3.90	6.71	2.81	8.15	4.25	5.22	1.32	10.75	6.85	7.37	3.47	6.94	3.04	6.80	2.90	7.08	3.18	6.50	2.60	7.20	3.30	7.99	4.09	8.10	4.20
Sept., 1902	4.50	5.47	0.97	5.10	.60	4.50	0	3.98	−.52	3.36	−1.14	4.70	.20	4.15	−.35	4.73	.23	3.97	−.53	4.50	0	6.64	2.14	6.24	1.74
Oct., 1908	1.25	2.58	1.33	3.88	2.43	4.25	3.00	4.46	3.21	0.40	−.85	.49	−.76	.50	−.75	.50	−.75	.48	−.77	.46	−.79	.23	−1.22	.55	−.70
Nov., 1905	6.45	2.40	−4.05	3.58	−2.87	.92	−5.53	3.30	−3.15	6.14	−.31	3.42	−3.03	5.00	−1.45	3.47	−2.98	4.78	−1.67	4.75	−1.70	4.77	−1.68	5.26	−1.19
Dec., 1907	4.51	4.48	−.03	4.28	−.23	2.50	−2.01	3.34	−1.17	6.12	1.61	5.17	.66	4.10	−.41	5.27	.76	3.92	−.59	5.16	.65	4.75	.24	4.70	.19
Year	54.31	58.54	+4.23	59.55	+5.24	56.62	2.31	59.15	4.81	49.62	−4.69	49.18	−5.13	50.35	−3.96	49.76	−4.55	50.70	−3.61	49.96	−4.35	60.03	+5.72	57.45	+3.14
Mean			1.54		1.52		2.41		2.10		1.24		1.08		.83		1.07		.95		1.09		1.22		1.11
Interpolated precipitation at North Bloomfield, Calif., from Bowman's Dam, Blue Canyon, and Nevada City records, 1899-1908.																									
Jan., 1904	3.85	11.38	7.53	10.12	6.27	9.60	5.75	12.52	8.67	[b] 5.37	1.52	4.31	+0.46	4.60	0.75	3.58	−0.27	3.69	−0.16	3.47	−0.38	2.23	−1.62	2.41	−1.44
Feb., 1905	7.05	10.07	3.02	9.72	2.67	13.19	6.14	8.16	−1.11	8.97	1.92	7.76	.71	8.35	1.30	6.60	−.45	6.70	−.35	6.10	−.95	6.70	−.35	6.77	−.28
Mar., 1903	15.04	12.68	−2.36	2.36	−12.68	14.15	−.89	4.37	−10.67	17.32	2.28	15.07	.03	16.00	.96	8.53	−6.51	12.85	−2.19	12.41	−2.63	10.52	−4.52	10.94	−4.10
Apr., 1901	5.29	3.61	−1.68	4.05	−1.24	4.09	−1.20	1.88	−3.41	5.99	.70	5.27	−.02	5.35	−.06	3.83	−1.46	4.30	−.99	4.48	−.81	9.96	4.67	9.22	3.93
May, 1902	1.70	2.88	1.18	2.79	1.09	1.16	−.54	3.18	1.48	2.76	1.06	2.16	.46	1.50	−.20	1.80	.10	1.20	−.50	1.68	−.02	2.17	.47	2.11	.41
June, 1905	0.22	1.29	1.07	2.10	1.88	2.16	+1.94	1.80	1.58	1.72	1.50	.79	.57	1.30	1.08	.60	.38	1.04	.82	.56	.34	.69	.47	.61	.39
July, 1905	0	0.12	.12	2.11	2.11	0	0	.04	.04	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Aug., 1901	T.	.83	.83	2.29	2.29	.02	.02	.27	.27	.05	.05	.01	.01	.05	.05	.01	.01	.04	.04	T.	0	.69	.69	.03	.03
Sept., 1907	0.15	1.29	1.14	.48	.33	.31	.16	.23	.08	.17	.02	.07	−.08	.20	.05	.42	.27	.16	.01	.07	−.08	.06	−.09	.05	−.10
Oct., 1906	0.14	4.50	4.36	1.98	1.84	.56	.42	2.30	2.16	.79	.65	.48	.35	.60	.46	.38	.24	.48	.34	.82	.68	.40	.26	.40	.26
Nov., 1902	6.09	6.50	.41	6.24	.15	9.98	3.89	5.52	−.57	10.04	3.95	8.03	1.94	8.75	2.66	6.71	.62	7.03	.94	6.97	.88	6.61	.52	6.58	.49
Dec., 1903	3.80	8.23	4.43	4.56	.76	8.02	4.22	3.83	.03	6.19	2.39	4.19	.39	5.05	1.25	3.52	−.27	4.05	.25	3.54	−.26	1.64	−2.16	1.50	−2.30
Year	43.33	63.38	20.05	48.80	+5.47	63.24	19.91	44.10	+0.77	59.37	+16.04	48.15	+4.82	51.75	+8.42	35.99	−7.34	41.54	−1.79	40.10	−3.23	41.67	−1.66	40.62	−2.71
Mean			2.34		2.78		2.20		2.51		1.38		.42		.74		.89		.55		.59		1.33		1.14

[a] Abbeville record

[b] Bowman Dam record.

TABLE 2.—*Comparison of methods of interpolating missing monthly rainfall records.*

Method of interpolation.	Dannemora, N. Y.	New Iberia, La.	T. S. Ranch, Colo.	North Bloomfield, Calif.	Average for the four stations.
(1)	(2)	(3)	(4)	(5)	(6)
Annual precipitation=total of the 12 months used	32.55	54.31	11.55	43.33	35.44
Average monthly precipitation	2.71	4.52	.96	3.61	2.95
Average error per month in inches.					
1-*a*. Normal	1.20	1.54	.50	2.34	1.40
1-*b*. Average of preceding and following months	1.36	1.52	.65	2.78	1.58
1-*c*. Average same month preceding and following years	1.30	2.41	.54	2.20	1.61
1-*d*. Angot method	1.30	2.10	.65	2.51	1.64
2-*a*. Nearest station	.79	1.24	.47	1.38	.97
2-*b*. Average of three surrounding stations	.43	1.08	.50	.42	.61
2-*c*. Inclined plane method	.55	.83	.41	.74	.63
3-*a*. Fournie method	.43	1.07	.44	.89	.71
3-*b*. Fournie-Horton method	.56	.95	.41	.55	.62
3-*c*. Abnormality method	.40	1.09	.42	.59	.63
3-*e*. Angot-Horton method	.76	1.22	.75	1.33	1.02
3-*f*. Angot-Leach ratio	.75	1.11	.62	1.14	.90
Mean	.78	1.35	.53	1.41	1.02
Average algebraic error.[a]					
1-*a*. Normal	0.09	0.35	0.09	1.67	0.55
1-*b*. Average of preceding and following months	.01	.44	.05	.46	.24
1-*c*. Average same month preceding and following years	.15	.19	.18	1.66	.54
1-*d*. Angot method	.12	.40	.12	.06	.18
2-*a*. Nearest station	.74	.39	.17	1.34	.66
2-*b*. Average of three surrounding stations	.37	.43	.22	.40	.36
2-*c*. Inclined plane method	.48	.33	.01	.70	.38
3-*a*. Fournie method	.31	.38	.09	.61	.35
3-*b*. Fournie-Horton method	.11	.30	.13	.15	.17
3-*c*. Abnormality method	.10	.36	.03	.27	.19
3-*e*. Angot-Horton method	.08	.48	.31	.14	.25
3-*f*. Angot-Leach ratio	.07	.28	.20	.23	.19
Mean	.22	.36	.13	.66	.34

[a] One-twelfth of the annual or total algebraic error.

The more refined methods, combining the data for the base and interpolation stations and including correction for local variation, give the most accurate results, especially with reference to reduction of the constant errors. Since the accuracy of interpolation varies with the amount and variability of the rainfall, the relative errors differ for the different stations, as shown in Table 4, which gives the average of all of the arithmetic and algebraic errors of 144 interpolations for each of the four interpolation stations.

Since all the different methods are not readily carried in mind, a brief statement of each, together with an estimate of the comparative labor involved in its use and the comparative accuracy of the results is given in Table 5. The comparative labor involved is estimated approximately on the basis of the time in minutes required to make a single interpolation when all the data, including normals, if any are required, have been compiled and are directly available.

In comparing the merits of different interpolation methods, three things are to be considered:

1. Labor involved.
2. Arithmetic error of individual interpolated values.
3. Total or average algebraic error of interpolated values. For a single or limited number of interpolations, simplicity and relative accuracy of the individual interpolations are most important. Where many interpolations are to be made, the constant coefficient methods, Fournie's, the abnormality, and the correction ratio methods become relatively much less laborious than where they are used for a small number of interpolations, since the coefficients or weights when once computed for a given group of stations can be used for all interpolations involving the same stations.

TABLE 3.—*Summary of rainfall interpolation methods—average results by different methods for all four groups of stations.*

Method.	Average monthly error.		Total average algebraic error.	
	Inches per month.	Per cent of true value.	Inches per month.	Per cent of average month precipitation.
(1)	(2)	(3)	(4)	(5)
Methods using same station only.				
1-*a*. Normal	1.40	62.7	±0.55	18.7
1-*b*. Preceding and following month	1.58	140.6	.24	8.1
1-*c*. Same month preceding and following years	1.61	108	.55	18.7
1-*d*. Angot method	1.64	92	.18	6.1
Average	1.56	100.8	.38	12.9
Contemporary record methods.				
2-*a*. Nearest station	.97	67.7	.66	22.4
2-*b*. Average of three surrounding stations	.61	43.8	.35	11.9
2-*c*. Inclined plane	.63	47.7	.38	12.9
Average	.74	53.1	.46	15.7
Combined methods.				
3-*a*. Fournie	.71	40.8	.35	11.9
3-*b*. Fournie-Horton	.62	41.2	.17	5.8
3-*c*. Abnormality	.63	40.4	.19	6.4
3-*e*. Angot-Horton method	1.02	50.4	.25	8.5
3-*f*. Angot-Leach ratio	.90	47.6	.19	6.4
Average	.78	44.1	.23	7.8

Average annual precipitation of the four stations 35.44
Average monthly precipitation of the four stations 2.95

Column 2=Average monthly error in inches. (Column 6, Table 2.)

Column 3=Average of the individual errors in per cent. Percentage for each month=error/true value.

Column 4=Average algebraic error=average annual error/12.

Column 5=Column 4 expressed as percentage of average monthly rainfall at all stations=column 4/2.95.

TABLE 4.—*Average results of all methods of rainfall interpolation methods at each of the four stations used.*

Station.	Average monthly precipitation.[a]	Average arithmetic error.		Average of the algebraic errors.	
		Inches per month.	Per cent per month.	Inches per month.	Per cent per month.
(1)	(2)	(3)	(4)	(5)	(6)
Dannemora, N. Y.	2.71	0.78	34.2	±0.22	8.1
New Iberia, La.	4.52	1.35	43.6	.36	8.0
T. S. Ranch, Colo.	.96	.53	77.9	.13	13.5
North Bloomfield, Calif.	3.61	1.41	105.2	.66	18.3

[a] Average of the 12 months used.

Column 3=Arithmetic average of the individual errors disregarding sign.

Column 4=Average of the individual per cent errors.

Column 5=Arithmetic average of one-twelfth of the annual algebraic error for each method.

Column 6=Column 5 expressed as per cent of average monthly precipitation, column 2.

TABLE 5.—*Summary of monthly rainfall interpolation methods.*

Group and method.	Description.	Other stations required.	Normals required.	Constant error.	Relative labor.[a] Few cases.	Relative labor.[a] Many Cases.	Average monthly error, per cent. Arithmetic.	Average monthly error, per cent. Algebraic.	Average monthly error, per cent. Sum of (8) and (9).
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1-a	Normal: Substitution of mean for same month	No.	1	No.	1	1	62.7	18.7	81
1-b	Use of mean of preceding and following months	No.	No.	Yes.	3	3	140.6	8.1	149
1-c	Mean of same month preceding and following years	No.	No.	Yes.	3	3	108	18.7	127
1-d	Angot method: Normals used with means of preceding and following months. $d=\frac{D}{D_1+D_2}\times\frac{d_1+d_2}{2}$	No.	3	No.	8	4	92	6.1	98
2-a	Use of nearest station record for same month	No.	No.	Yes.	1	1	67.7	22.4	90
2-b	Average of three surrounding stations: $d=\frac{a+b+c}{3}$	3	No.	Yes.	3	3	43.8	11.9	60
2-c	Inclined plane, with three surrounding stations	3	No.	Yes.	10	10	47.7	12.9	61
3-a	Fournie method: $d=\frac{1}{3}\left(\frac{D}{A}a+\frac{D}{B}b+\frac{D}{C}c\right)$	3	4	No.	20	10	40.8	11.9	53
3-b	Fournie-Horton: A ratio deduced from the three Fournie values $\frac{D}{A}$, $\frac{D}{B}$, $\frac{D}{C}$ by the inclined plane method is applied to the value of d deduced directly by the inclined plane method	3	4	No.	40	20	41.2	5.8	47
3-c	Abnormality method: $d=\frac{1}{3}\left(\frac{a}{A}D+\frac{b}{B}D+\frac{c}{C}D\right)$. Identical with Fournie method, but computed differently	3	4	No.	20	10	40.4	6.4	47
3-d	Correction-Ratio method: Uses contemporary data for three surrounding stations. $d=\frac{1}{3}\left(\frac{a}{a_1+a_2}+\frac{b}{b_2+b_2}+\frac{c}{c_1+c_2}\right)(d_1+d_2)$	3	No.	No.	20	20	(b)	(b)	....
3-e	Horton method: Same as preceding, but correction factor applied to d_1+d_2 obtained by applying inclined plane method to the three ratios	3	No.	No.	30	30	50.4	8.5	59
3-f	Leach method: Same as preceding, except that the three ratios are weighted by inverse distances from base station	3	No.	No.	30	20	47.6	6.4	54

Notation A, B, C, D, normals at base and interpolation stations, respectively; a, b, c, d, monthly and interpolated values, subscripts 1 and 2 relate to preceding and following months.

[a] In estimating relative labor, it is assumed that normals and other data are directly available without computation.

[b] Not determined.

Independent of the labor involved, the value of a method may be considered as about proportional to the sum of the average and algebraic errors resulting from its use. This sum is shown in column (10) of Table No. 5. Where many interpolations are required it becomes important that the total or algebraic mean error of all interpolations should be small or that the algebraic errors should tend to vanish as the number of interpolated values increases. In the case of certain methods the algebraic error may be cumulative, there being a constant difference involved between the interpolated and true values. This may result in using (*a*), the nearest station; (*b*), the mean of three surrounding stations; (*c*), the mean of the preceding and following months. Where the interpolation months are scattered equally throughout the year there should be no tendency for a cumulative algebraic error in using the last-named method. For stations having relatively small precipitation, as, for example, North Bloomfield, Calif., the percentages of errors are misleading. A percentage of error of even 1,000 per cent for a month with a precipitation of only 0.01 inch may be of little importance hydrologically, whereas a 10 per cent error for a monthly precipitation of 10 inches would be of much greater significance. On the other hand, actual errors taken alone may be misleading, since actual errors in inches are likely to be smaller with stations for small than for stations with large precipitation.

The simplest methods, Groups 1 and 2, involve relatively little labor, but the errors are comparatively large. As regards accuracy, the mean of three surrounding stations and the inclined plane method are decidedly the best in these groups. As between the direct use of the average of three surrounding stations and the application of the inclined plane method, the results given in the table show little choice. It appears certain, however, that inasmuch as the inclined plane method gives weights to the surrounding stations, decreasing as their distances from the interpolation station increase, this method if applied to a sufficient number of cases to give a decisive result would show greater accuracy than the simple average for three surrounding stations. While the labor involved in the use of the inclined plane method is somewhat greater than where the simple average of three stations is used, yet it is comparatively slight in any event, and the use of the inclined plane method in preference to the simple average of three stations seems advisable.

With reference to the methods of Group No. 3, the differences in accuracy are not very large. All methods of Group 3 show materially smaller algebraic errors than the simpler methods. The choice between the methods of Group 3 must apparently, therefore, depend largely on the labor involved. There is no very great difference between the methods of Group 3 when the data have once been compiled. In the case of methods involving the use of normals, viz., Fournie, Fournie-Horton, and Angot methods, the actual labor for a small number of interpolations will often be much greater than the relative labor indicated by the table, especially if the normals themselves are not available without computation. If, therefore, results substantially as accurate as those obtained by the use of methods involving normals can be procured without the use of normals, then the methods avoiding the use of normals are generally to be preferred. The Horton and Leach methods do not involve the use of normals but are based entirely on contemporaneous records. They take into account the relative positions of the stations and utilize data for the base as well as for surrounding stations, and as shown by Table 5, the accuracy of these methods is nearly equal to that obtained by the use of methods involving normals. As between the Horton and Leach methods, the advantage appears to lie generally with the latter, both in point of accuracy and in simplicity of application, especially where many interpolations are made, since the ratios once determined for the Leach method for a given month and group of stations can be applied to other interpolations at the same station for the same month of the year. If two of the base stations happen to lie nearly in line with the interpolation station, then the inclined plane method gives comparatively little weight to the third base station. Under such conditions the method of weighting by inverse distances, used in the Leach method, is preferable. Inasmuch as the studies thus far made indicate that

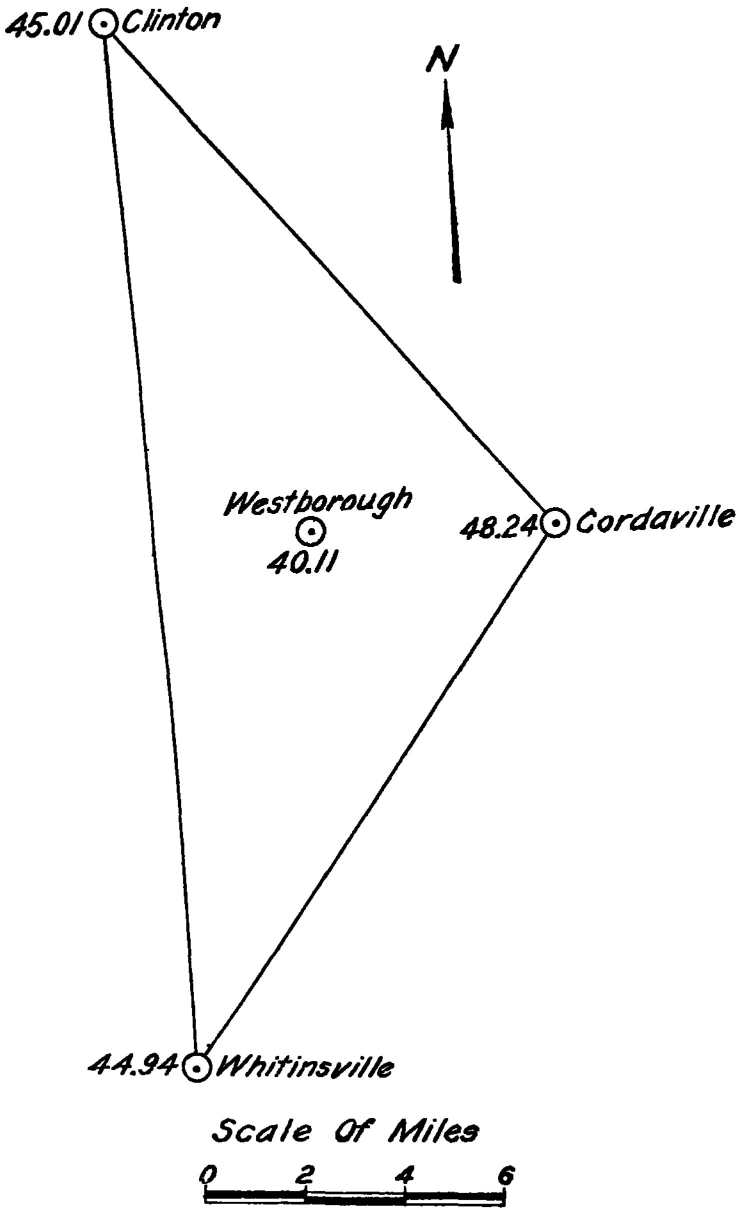

FIG. 7.—Location of rainfall stations, Massachusetts group. (*Figure gives mean annual rainfall.*) 1895–1908, inclusive.

nearly as great accuracy can be obtained by the use of the Horton and Leach methods without the use of normals as from those methods involving normals. A further study was made to test the applicability of these two methods to localities of moderately variable rainfall. Groups of stations were chosen in Massachusetts, Michigan, and Nebraska, Figures 7, 8, and 9. Interpolations first were made for a total of 10 months for each station by using the mean of three surrounding stations. The results are given in the first line of each section of Table No. 6.

TABLE 6.—*Comparison of methods 2–b, 3–e, and 3–f for the interpolation of rainfall records.*

	Method.	West-boro, Mass.	Battle Creek, Mich.	Ashton, Nebr.	Mean.
Actual precipitation 10 months (inches)	....	31.57	32.63	20.30	....
Total of 10 monthly interpolations (inches)	2–b	32.70	32.62	19.74	....
	3–e	30.61	42.32	22.63	....
	3–f	31.41	38.48	20.23	....
Algebraic error of 10 monthly interpolations inches	2–b	1.19	−.01	−.56	0.21
	3–e	−.90	9.69	2.33	3.71
	3–f	−.10	5.86	−.07	1.90
Arithmetic average monthly error of interpolation (inches)	2–b	.407	.627	.51	.51
	3–e	.456	1.615	.623	.90
	3–f	.424	1.465	.435	.77
Average monthly percentage of error of interpolation	2–b	15.7	29.7	77.8	41.1
	3–d, e	15.3	63.6	51.7	43.5
	3–e, f	14.5	57.0	45.8	39.1
Number of plus errors	2–b	6	5	6	....
	3–e	5	7	7	....
	3–f	5	6	7	....
Number of minus errors	2–b	4	5	3	....
	3–e	5	3	2	....
	3–f	5	4	2	....

2–b= Average of the three nearest stations.
3–e=Corrected ratio to the preceding and following months.
3–f= Weighted (inversely as distance) corrected ratio to preceding and following months.

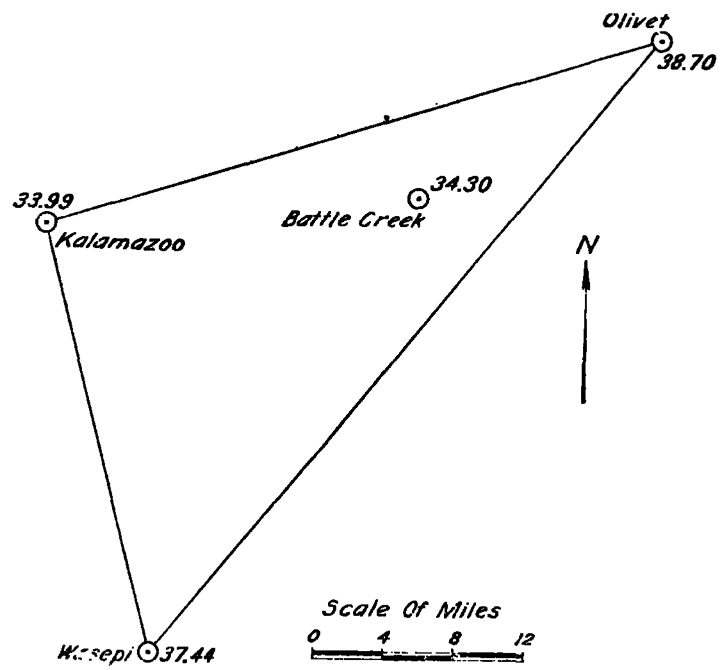

FIG. 8.—Location of rainfall stations, Michigan group. (Figure gives mean annual rainfall.) 1895–1908, inclusive.

In this series the average arithmetic error of the mean of three surrounding stations is 41.1 per cent, which is not materially different from the value 43.8 per cent obtained for the first series of stations. The average monthly arithmetic error of the Horton and Leach methods are, respectively, 43.5 and 39.1 per cent in the second series, again showing slightly better results for the Leach method. The average percentage of error among the interpolations in the second series by both methods is several per cent less than in the first series, showing that better results are obtained by these methods in regions of moderate than in regions of high rainfall variability. It will be noted that in the second series the Horton and Leach methods do not give quite as small an average monthly percentage of error as is obtained from the direct average of three surrounding stations. The same was true in the first series. Again, the Horton and Leach method give larger algebraic monthly errors in the second series than does the simple average of three surrounding stations. This, however, apparently results almost entirely from an accidentally small error of the average of three surrounding stations for the interpolations at Battle Creek, and in the first series the average monthly algebraic error of the Horton and Leach methods was materially less than for interpolations based on the direct average of three surrounding stations.

While the number of sample compilations compared in these studies is obviously insufficient for a final determination of the relative accuracy of different methods, yet the following tentative conclusions appear to be justified. In this connection it should be borne in mind that the arithmetic error indicates the probable departure of a single interpolated value from the true value. The algebraic error shows the tendency for

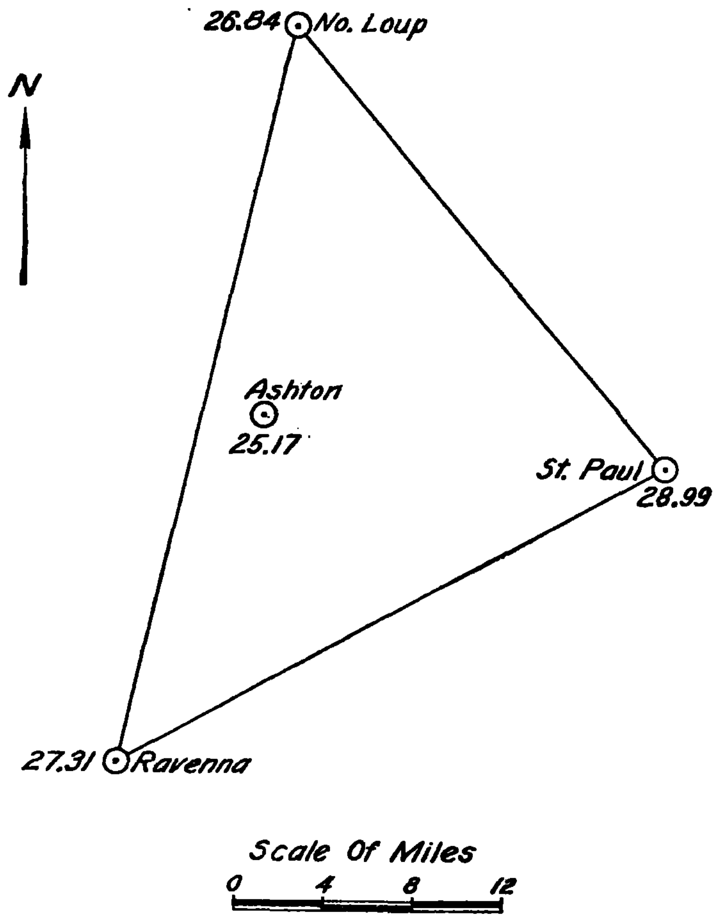

FIG. 9.—Location of rainfall stations, Nebraska group. (Figure gives mean annual rainfall.) 1895–1908, inclusive.

the occurrence of set or cumulative errors. If the monthly errors tend to counterbalance, the algebraic error will be small. The sum of the two errors here used as an utility index has no statistical meaning, except that if it is large at least one of the errors is necessarily large.

INTERPOLATION OF ANNUAL RAINFALL AND REDUCTION TO BASE PERIOD.

The preceding discussion has been devoted to interpolation of missing monthly rainfall amounts. The same methods can of course be applied to filling in missing seasonal or annual values. In general, since annual is less variable than monthly rainfall, the accuracy of the result will be greater for annual than for monthly rainfall. It is important in preparing maps of average rainfall or in determining the mean rainfall on a drainage basin to reduce all the records used to a uniform base period. If some of the records available do not cover the entire base period a suitable method should be used to reduce these records so as to obtain the approximate average rainfall at the same stations for the base period. The simplest procedure is the use of the direct ratio method, often used in Europe and attributed to Hugo Meyer[5] in which the derived average is

$$P' = \frac{P}{a} X A = \frac{A}{a} X P \qquad (1)$$

where A is the base period average at an adjacent station, a is the average at the same station for the period covered by records at both the base and interpolation stations, P is the mean precipitation at the interpolation station for the period of record. It will be seen that this is identical with the Fournie method already described, except that in the latter, three base stations are used. Designating the other two stations B and C, and using notation similar to above, the average precipitation for the base period at the interpolation station is obtained by the formula,

$$P' = \frac{P}{3}\left(\frac{A}{a} + \frac{B}{b} + \frac{C}{c}\right) \qquad (2)$$

This is an excellent method of reduction to a base period and is the one most generally used in the United States. It is more rational and probably more accurate in most cases to apply the inclined plane method to the ratios $\frac{A}{a}$, $\frac{B}{b}$ and $\frac{C}{c}$. Then, calling the resultant ratio for the location of the interpolation station r', the resulting base period average for the interpolation station is,

$$P' = r'\,P \qquad (3)$$

Recently Von P. Heidke[6] described a method of extension of short rainfall records based on the method of least squares. Several stations are used as in the Fournie method. They are given weights inversely proportional to the squares of the mean errors of the reduced means for the individual stations. First, trial values of the rainfall P' for the interpolation station are computed by the use of Meyer's ratio for each base station. Calling these trial values P'_a, P'_b, P'_c, etc., and the weights to be applied to them w_a, w_b, w_c, etc., the base period precipitation at the interpolation station is obtained by means of the formula,

$$P' = \frac{w_a P'_a + w_b P'_b + w_c P'_c + \text{etc.}}{w_a + w_b + w_c + \text{etc.}} \qquad (4)$$

The true mean errors of the reduced means derived from the several base stations can not of course be determined in advance, since the true base period precipitation at the interpolation station is unknown. Heidke, however, assumes that the true mean errors for the several base stations are proportional to the corresponding mean errors of the precipitation amounts P'_a, P'_b, P'_c, etc., derived by the use of the Meyer ratios for the several base stations. Calling these mean errors m_a, m_b, etc.,

[5] Anleitung zur Bearbeitung meteorologischer Beobachtungen für die Klimatologie, Berlin, 1891.

[6] Reduktion kürzerer Reihen von Niederschlagsmessungen auf die langjährigan homogener Nachbarstationen unter Berücksichtigung von Gewichten,—*Meteorologische Zeitschrift*, June, 1923, pp. 167–173.

the corresponding weights to be applied in formula (4) are,

$$w_a = \frac{c}{m_a^2},\ w_b = \frac{c}{m_b^2},\ \text{etc.} \tag{5}$$

where c either equals unity or may be given an arbitrary value, say 100, more convenient for computation purposes.

The following table (7) illustrates the method of computation of the weight w for Rehoboth to be used in determining the long term average precipitation from a short record at Mariental. Column (2) shows the available precipitation date at Mariental and column (3) shows the corresponding precipitation at Rehoboth. The ratio of the means, including two incomplete record years, is 0.753. Using this value of the Meyer ratio the trial values of precipitation at Mariental for the same years are computed as shown in column (6). The departures of these trial values from the true values are next determined, as shown in column (7), and their squares taken, as given in column (8). The mean error of the trial values is determined by the formula,

$$m = \sqrt{\frac{\Sigma(\Delta^2)}{n}} \tag{6}$$

where n is the number of years of complete record, which is nine in this case. This leads to a value 0.12 for the weight to be applied to Rehoboth base station. Proceeding in a similar manner weights to be applied to other base stations are determined.

TABLE 7.—*Computation of weight, Heidke's method.*

Year.	Precipitation		$P/a = \frac{(2)}{(3)}$	Departure of (2) from mean = (2) − 146.	0.753 × (3).	(2) − (6)	$(7)^2$
	Mariental = P.	Rehoboth = a.					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Mm.	*Mm.*		*Mm.*	*Mm.*	*Cen.*	*Cen.*
1899-1900	[a] 189	193	......	......	145	4	......
1900-1901	102	167	0.61	−44	126	−2	4
1901-02	86	123	0.70	−60	93	−1	1
1902-03	132	111	1.19	−14	84	5	25
1903-04	286	398	0.72	140	300	−1	1
1907-08	[a] 177	199	......	......	150	3	......
1908-09	234	343	0.68	88	258	−2	4
1909-10	172	257	0.67	26	194	−2	4
1910-11	56	106	0.53	−90	80	−2	4
1911-12	152	262	0.58	6	197	−4	16
1912-13	97	76	1.28	−49	57	4	16
Σ	1,683	2,235	......	......	1,684	+16	75
Average	[b] 146	[b] 205	0.77	57.4	......	−14	......

Ratio of means $= \frac{1,683}{2,235} = 0.753 = r_1$

$$m = \sqrt{\frac{\Sigma(\Delta^2)}{n}} = \sqrt{\frac{75}{9}} = 29 \text{ cen.} = 290 \text{ mm.}$$

$$w = \frac{100}{29^2} = 0.12$$

[a] Incomplete year, interpolated.
[b] Average for nine complete years.

This method involves two assumptions:

(1) That the departures of individual rainfall amounts from the average behave as normal errors.

(2) That the true mean errors of the rainfall at the interpolation station are porportional to the trial mean errors determined from Meyer's ratios.

To test the first assumption, Heidke uses the criterion that for normal errors.

$$\frac{2n\,[\Sigma\,\Delta^2]}{\Sigma \pm \Delta} = \pi$$

Calling E_1 the true value of the left-hand member of equation (6), derived from two simultaneous long-term records, and calling E_2 the approximate value derived from the use of Meyer ratios, Heidke obtains by a comparison of five pairs of stations having 50-year records, average values of E_1 and E_2 as follows: $E_1 = 3.07$, $E_2 = 3.21$. Similarly, from a comparison of 30-year records for five pairs of stations he obtains: $E_1 = 2.96$, $E_2 = 3.15$. From five pairs of 20-year records he obtains: $E_1 = 2.86$, $E_2 = 3.09$. These are to be compared with the theoretical value, $\pi = 3.1416$.

It should be noted that the frequency curve of annual precipitation at a given station is not a true normal error curve but is somewhat skewed, as evidenced by the well-known fact that there are more dry than wet years. Nevertheless the variation from the normal or Gaussian law of error is probably not sufficient in most cases to vitiate the utility of Heidke's method. The method is, however, laborious, and it is doubtful whether the increased accuracy, if any, obtained by its uses as compared with the Fournie or Fournie-inclined plane method will justify its application except in cases where the utmost possible accuracy is required.

CONCLUSIONS.

1. The average arithmetic error of monthly rainfall interpolations may exceed 100 per cent of the true rainfall where the simpler methods of interpolation involving data for the base station only are utilized, but the error can generally be reduced to 50 per cent or less by the use of three-station methods.

2. The use of the direct three-station average and the inclined-plane method give nearly as small an arithmetic error as any methods and are the simplest to apply of the more accurate methods. Of these two the inclined-plane method is the more rational and probably the more dependable.

3. The direct three-station average and inclined-plane average give only about one-half as large algebraic errors as single-station methods. In regions of low to moderate rainfall variability the average algebraic error of these methods are no larger than those for the more refined methods.

4. Because of their simplicity and the avoidance of the labor of using normals, the direct three-station average and inclined-plane methods are the ones best adapted for use in regions of low or moderate rainfall variability.

5. In regions of high rainfall variability the arithmetic, algebraic, and total percentage errors are generally the least for interpolation methods involving the use of Fournie ratios and normals.

As nearly as good results can, however, be obtained from the use of contemporaneous records only by means of weighted correction ratios as applied in the Horton and Leach methods. Considering the reduced labor usually involved in the application of these methods through the avoidance of using normals, they appear to be the methods best adapted for regions of high rainfall variability. Of the two, the Leach method is apparently somewhat the better.